

# Finite element design of concrete structures

G.A. Rombach

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Practical problems and their solutions

G. A. Rombach

 Thomas Telford

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# Foreword

Over the last few years electronic data processing has changed the work of engineers throughout all fields of construction. This is particularly true for design of structures where, nowadays, it is unimaginable that it can be done without the help of computer software. Even simple structures like, for example, simply supported reinforced concrete beams under uniform loading, are designed using the help now available from computers. One must admire this. In many cases, these computer calculations are faster, less costly and thus more profitable than manual calculations.

The developments of the last year or so have been to use yet more complex numerical models, as can be seen from the various contributions to conferences on the subject. It seems that today, it is only a question of computer capacity, the size of the element mesh, and the modelling of the nonlinear material behaviour, in order to model an arbitrary complex reinforced structure with almost unlimited accuracy. But then, there is a great danger that one only believes in the results from the computer, and the engineer loses his feelings for the real behaviour of the structure. Thus, in this book, the author faces up to the blind belief in computer results. One should not have a totally blind confidence, but rather a useful scepticism of the output of the computer calculations, regarding the numerical model used and hence the results achieved.

With the increasing complexity of a numerical model, it becomes more likely that important details are overlooked, due to the flood of information produced by the computer. The collapse of the so-called Sleipner Platform (see Chapter 1), resulting from an erroneous Finite Element calculation, impressively demonstrates this danger.

A complex numerical calculation should not be used to compensate for any lack of knowledge of the structural behaviour of a structure. An

engineer should be able to simplify any real structure into a well-defined, known, understandable, and designable equivalent structural system. Unimportant details are neglected. It should always be kept in mind that even very complex structures such as the Chapel of St Peter's Church in Rome or the Temples in Luxor, had been built without the help of computers and even without any knowledge of mechanics.

This book has been written for both the practical structural engineer and for students, who are using computer software for designing of concrete structures. The problems of Finite Element calculations are illustrated, not just by theoretical systems, but also by relating to real structures, mostly those on which the author has actually worked. They concern systems from all fields of engineering. Furthermore, this book should help those people who develop software for structural design to understand the difference between theory and the daily problems of designing reinforced concrete structures.

This book would not have been written without the help and support of friends and colleagues in practice and research. I am much indebted to Peter Whiting LL B (Hons), BSc. FICE for reviewing of the manuscript and his support of my work.

*Guenter Axel Rombach*  
Hamburg, 2004



# Notations

In general the symbols of Eurocode 2 are used. These are listed below together with additional abbreviations used in this book.

## 1 Latin upper-case letter

$A$	Accidental action; cross-sectional area
$A_c$	Cross-sectional area of concrete
$A_p$	Area of a prestressing tendon or tendons
$A_s$	Cross-sectional area of reinforcement
$A_{s,min}$	Minimum cross-sectional area of reinforcement
$A_{s,prov}$	Area steel provided
$A_{s,req}$	Area steel required
$A_{sw}$	Cross-sectional area of shear reinforcement
$C$	Symbol for grade of normal concrete
$C_m$	Wrapping torsional stiffness
$E$	Effect of action (member force)
$E_c$	Tangent modulus of elasticity of normal weight concrete
$F$	Force; action
$F_d$	Design value of action
$FE$	Finite Elements
$G_k$	Characteristic permanent action
$H$	Horizontal force
$I$	Second moment of area
$L$	Length
$M$	Bending moment
$M_{Ed}$	Design value of the applied internal bending moment
$N$	Axial force
$N_{Ed}$	Design value of the applied axial force (tension or compression)

$P$	Prestressing force
$P_0$	Initial force at the active end of the tendon immediately after stressing
$P_{mt}$	Mean value of the prestressing force at time $t$ , at any point distance $x$ along the member
$Q_k$	Characteristic variable action
$R$	Resistance
$R_d$	Nominal value of resistance
$S$	Internal forces and moments
$S$	First moment of area
$S_m$	Centre of torsion of a cross-section
$S_s$	Centre of gravity of a cross-section
SLS	Serviceability limit state
$T$	Torsional moment
ULS	Ultimate limit state
$V$	Shear force

## **2 Latin lower-case letters**

$a$	Distance; geometrical data
$\Delta a$	Deviation of geometrical data
$a_1$	Shift of moment curve
$b$	Overall width of a cross-section, or actual flange width in a T- or L-beam
$b_{sup}$	Width of the support
$b_w$	Width of web on T, I, L beams
$c$	Concrete cover
$d$	Diameter; depth
$d$	Effective depth of a cross-section
$e$	Eccentricity
$f$	Strength (of a material)
$h$	Height
$h$	Overall depth of a cross-section
$i$	Radius of gyration
$l$	Length; span
$l_b$	Anchorage length
$l_{col}$	Height of a column
$l_{eff}$	Effective span of beams and slabs
$l_n$	Clear distance from the faces of the supports
$m$	Moment per unit length; mass
$n$	Number of vertical continuous members
$r$	Radius
$x$	

$s$	Distance; spacing of stirrups
$p$	Mean transverse pressure over the anchorage length
$t$	Time being considered; thickness
$u$	Perimeter of concrete cross-section having area $A_c$
$v$	Shear force per unit length
$\nu$	Coefficient relating the average design compressive stress in struts to the design value of the concrete compressive strength ( $f_{cd}$ )
$\nu$	Angle of inclination of a structure, assumed in assessing effects of imperfections
$x$	Neutral axis depth
$z$	Lever arm of internal forces

### 3 Greek letters

$\alpha$	Angle; ratio
$\beta$	Angle; ratio; coefficient
$\gamma$	Partial safety factor
$\delta$	Increment
$\zeta$	Reduction factor; distribution coefficient
$\varepsilon$	Strain
$\theta$	Angle; rotation
$\lambda$	Slenderness ratio
$\mu$	Coefficient of friction between tendons and their ducts
$\mu$	Moment coefficient
$\nu$	Poisson's ratio
$\nu$	Strength reduction factor for concrete cracked in shear
$\nu$	Longitudinal force coefficient for an element
$\xi$	Ratio of bond strength of prestressing and reinforcing steel
$\rho$	Over-dry density of concrete in $\text{kg/m}^3$
$\rho$	Reinforcement ratio
$\sigma$	Normal stress
$\sigma_c$	Compressive stress of concrete
$\sigma_s$	Tensile stresses in reinforcement
$\tau$	Torsional shear stress
$\varnothing$	Diameter of a reinforcing bar or of a prestressing duct
$\varphi(t, t_0)$	Creep coefficient, defining creep between times $t$ and $t_0$ , related to elastic deformation at 28 days
$\Psi$	Factors defining representative values of variable actions
	$\Psi_0$ for combination values
	$\Psi_1$ for frequent values
	$\Psi_2$ for quasi-permanent values



**4 Subscripts**

c	Concrete; compression; creep
b	Bond
d	Design
e	Eccentricity
eff	Effective
f	Flange
fat	Fatigue
fav	Favourable
freq	Frequent
g	Permanent action
h	Depth of a cross-section
i	Indices; notional
inf	Inferior; lower
j	Indices
k	Characteristic
l	Low; lower
m	Mean; material; bending
max	Maximum
min	Minimum
nom	Nominal
p	Prestressing force
perm	Permanent
pl	Plastic
q	Variable action
rep	Representative
s	Reinforcing steel; shrinkage
sup	Superior; upper
t	Torsion; time being considered; tension
unf	unfavourable
w	Web
y	Yield

# 1

## General

Numerical calculations based on the Finite Element Method are becoming a standard tool in the design of structures. Furthermore, the lower cost of hardware and increased performance of more user-friendly software often displace manual calculations. This applies not only to complicated 3-dimensional structures, like slabs, shear walls and shells of complicated shape, but also to normal beams. It can be economical – as it is much faster – to design a simple supported reinforced concrete beam under uniform loading by using a computer. However, one saves time only when the necessary checking of the numerical results are omitted.

A few years ago large computers were needed and only experts and big consulting offices used this method. Nowadays, a whole building can be handled by a simple PC. Graphical input makes it easy to generate three-dimensional Finite Element meshes with several thousands of nodes. Computer programs can design concrete, steel or wooden structures, which have linear or nonlinear material behaviour, under static or dynamic loading. There no longer seems to be any limitations. Nonetheless, this development has led to an increasing number of cases where the Finite Element Method has been misused.

As daily experience shows, results from computer calculations are often trusted with blind faith. Users assume that expensive computer software must be free from any error. A graphical pre-processor and a user-friendly input of systems and loadings may suggest a high technical competence and reliability of the computer program. Nevertheless, as experience in practice shows, this confidence can only be justified to a very limited degree. Almost no software is free from errors. Therefore, a critical distrust is appropriate, as program errors may also occur in software which has been in use for a long time and which may not have been found to date.



It should always be kept in mind that the Finite Element Method is only a numerical tool based on numerous assumptions and simplifications. This must be considered when using a software for design of structures. Otherwise, the result of the numerical calculations can be totally wrong. For explanation purposes, the following is a very simple example: a plate element only provides a numerical model of a real slab. It is assumed to have a linear strain distribution over its depth under pure bending. There are no stresses at the midplane. With such a plate element, one will never be able to estimate the normal forces of a simple supported rectangular slab due to temperature changes or shrinkage, even if the supports are fully restraint in horizontal directions.

The modelling, the discretization, of real reinforced concrete structures is the focal point of this book. The fundamental aspects are illustrated by practical examples of concrete structures. This book does not look into the fundamental basis of the Finite Element Method, as numerous publications are already available (see, for example, references [1–3]). The so-called state of the art of the Finite Element techniques will not be discussed as there seems to be a great gap between the ongoing research and the day-to-day problems that a structural engineer has to face. An engineer has neither the time to make highly sophisticated numerical models nor the experimental data to verify his analysis. He is not even interested in the 'correct' results. His goal is simply to estimate the required amount of reinforcement and its accurate arrangement (the 'dimensioning' of a structure), in order to build a safe and economical structure. The calculation of the member and internal forces and moments is only a required step to reach this goal.

The examples shown in this book are calculated using standard software, used in day-to-day practice, and not with one of the advanced general-purpose Finite Element packages such as ABAQUS, ADINA or ANSYS which offer a great variety of different elements and material models. Hence, the reader can easily verify the given examples using his or her own software. A further reason for the strong relation to practical design is that a user of a software package is not usually familiar with its theoretical background. He cannot modify it. It does not help him if he knows that, for example, a reduced integrated 3-noded shell element may give better results than a full integrated 6-noded isoparametric element. He is just using the 'black box'. The user, however, is supposed to have sufficient knowledge to see and solve the problems, which may occur in a Finite Element analysis. This is where this book is intended to provide help.



It is surprising that, in structural engineering, the use of Finite Elements causes numerous problems, especially as this numerical method was first used by structural engineers. The world's first electronic programmable calculator was built by the *structural* engineer Konrad Zuse in May 1941. He was tired of repeating calculation procedures when designing structures. Zuse also developed the first algorithmic programming language, 'Plankalkuel'. In other fields of engineering, like, for example, the automobile or aircraft industry, the numerical FE-analysis of highly complex problems, such as the crash behaviour of a car, the optimization of aerodynamics, or the processes in the engine, have become a day-to-day practice. The reasons for this discrepancy is that these sorts of costly and complicated computer calculations are only economical for mass products. In contrast, a building is usually a unique structure, whose costs depend on several factors, not only the cost of its building material. The numerical modelling of the complex behaviour of the composite material 'reinforced concrete' causes far greater problems than the elasto-plastic bilinear behaviour of metals.

This book focuses on the numerical analysis of structures made of reinforced or prestressed concrete. Finite Element calculations of concrete structures have the following different and exceptional features in comparison with other materials:

- Reinforced or prestressed concrete is a composite inhomogeneous material with a very complex nonlinear material behaviour, thus an 'exact' model is far too costly for daily structural design. Therefore, the calculations of the member forces are mostly based on a linear elastic material model. Stiffness reductions, as a result of crack formations or the 'yielding' of concrete in regions with high compressive stresses, are ignored. The extent of this very large simplification should be justified for each calculation.
- The required material parameters, like, for example, elastic modulus  $E_c$ , concrete compressive strength  $f_c$ , Poisson's ratio  $\nu$ , show a large scatter in comparison with other construction materials. Furthermore, they are often time-dependant. The actual quality of construction (workmanship, weather conditions, curing) is not known during the design stage, however, this can influence the material parameters.
- Concrete material is often used for massive members where, for example, the Bernoulli hypothesis of a linear strain distribution over the depth of the cross-section does not apply. Therefore, standard beam and plate elements should not be used for the design of the so-called discontinuity regions.