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Concrete Fracture Models and Applications

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Dedicated to All Those who Relentlessly
Endeavour to Unite a Fractured World

Foreword

Fracture mechanics as a discipline of mechanics goes back to the early years of the 20th century. It started with the description and explanation of the cracking behavior and failure of glass which could not be explained by means of the strength of materials approach. The material to which the new theory was applied had to be elastic and brittle. After glass, the failure of brittle types of metals was investigated. Later, linear elastic fracture mechanics was extended to elastic–plastic material behavior with well-established theories.

Although concrete exhibited brittleness in conventional force-controlled tensile tests, it was only in the early 1960s that fracture mechanics principles penetrated slowly into the field of concrete. First attempts were made to apply linear elastic fracture mechanics to concrete, but there was no great success. However, the idea to apply fracture mechanics to concrete and concrete structures was very important. Many researchers started to think of concrete and fracture mechanics. At the same time, the testing facilities developed enormously. It became possible to perform displacement-controlled tensile tests on concrete. One realized that concrete is not perfectly brittle but strain softening, i.e., failure in tension, occurred only after a considerable nonelastic displacement in the post-peak region. The idea of cohesive stresses in a concrete crack emerged.

After recognizing the real behavior of concrete, nonlinear fracture mechanics theories were developed. The present book presents these theories in detail. The book is a comprehensive treatment of the state of knowledge and adds some new findings to the field. The authors succeeded to write the book in a way that sometimes complicated theories can be followed with ease. I congratulate the authors to this achievement and wish that not only many teachers will use the book in their classes but also code makers will use it as a compendium of the principles of fracture mechanics of concrete in order to introduce these principles finally into the design standards.

Stuttgart, Germany
July 10, 2010

Hans-Wolf Reinhardt

Preface

Concept of linear elastic fracture mechanics was first applied to pre-cracked concrete elements in the early 1960s. Thereafter, extensive experimental and numerical research investigations proved that the classical form of linear elastic fracture mechanics cannot be applied to normal size concrete members. From the past research and studies it also became clear that the modified form of linear elastic fracture mechanics or nonlinear fracture mechanics can be useful and powerful tools for the analysis of the growth of distributed cracking and its localization in concrete if the softening behavior of the material is taken into account. The nonlinear fracture models coupled and introduced the tension-softening constitutive law in the fracture mechanics concepts to study the crack initiation and its propagation in concrete and concrete structures.

In due course of time, a number of nonlinear fracture models have been proposed and used to predict the nonlinear fracture behavior of cementitious materials. These are fictitious crack model or cohesive crack model, crack band model, two-parameter fracture model, size-effect model, effective crack model, K_R-curve method based on cohesive force distribution in the fracture zone, double-*K* fracture model, and double-*G* fracture model. Fracture mechanics concept introduced energy approach for crack development and its growth which can avoid the unobjectivity in the results, predict the post-peak response with a less complexity and exhibit size-effect behavior. The brittleness of the material can quantitatively be defined and more uniform safety of factors can be achieved in the structural design with the help of fracture mechanics concept.

It is a well-known phenomenon that the fracture parameters of concrete depend on the softening function of concrete, concrete strength, specimen size, specimen geometry, geometrical factors like relative size of notch length and the loading condition. The literature reports extensive numerical and experimental investigation on nonlinear fracture behavior of concrete. All the important nonlinear fracture models are widely applied to characterize the related fracture parameters and these studies are available in scattered literature. This book attempts to present the theoretical development and applications of various nonlinear concrete fracture models in a unified manner using different fracture parameters. In this regard, the authors investigated the behavior of fracture parameters of concrete at different phases of crack propagation phenomena of concrete.

There are six major chapters in the textbook which are mainly based on the recent research and studies carried out by the authors in the recent years. The detailed introduction of the book is mentioned in the opening chapter. In the subsequent chapters, cohesive crack model for three-point bending test, four-point bending test, and compact tension specimens using important softening functions of concrete are developed. The numerical results are compared with the experimental results available in the literature. Further, a systematic study on the different cohesive crack fracture parameters is carried out. Introduction of weight function method is explained to determine the double-K fracture parameters and the K_R-curve method based on cohesive stress distribution. Furthermore, attempts are made to put forward some new developments regarding behavior of different fracture parameters using the cohesive crack model as the reference. A comprehensive comparison between the double-K and double-G fracture criteria is presented. Emphasis on the effect of various parameters including specimen geometry, size effect, and loading condition on the double-K and the K_R-curve method is also focused. Finally, a comparative study among different fracture parameters obtained from important nonlinear models is presented. Hence, the textbook presents results of a comprehensive study on the crack initiation and its growth in concrete-like materials using various fracture models. At last, the flowcharts of various fracture models are presented in Appendix.

In this book, the authors have taken a small step to present a basic introduction on the various nonlinear concrete fracture models considering the respective fracture parameters. It can be helpful to undergraduate and postgraduate students who are studying this subject. An immense help to the beginners and researchers in the area of fracture mechanics of concrete is expected from this book which will provide a sound basis on the relevant subject to carry out further innovative research work in the future. Appendix can be of much use to the readers for computing different fracture parameters using computer programs.

At last, the authors would be thankful to the readers and their invaluable suggestions or comments for the further improvement of the book.

Jamshedpur, India Shailendra Kumar
Kharagpur, India Sudhirkumar V. Barai

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Experimental and numerical investigations showed that the classical form of linear elastic fracture mechanics (LEFM) cannot be applied to normal size concrete members because of the presence of large and variable size of fracture process zone. However, when the softening behavior of material was taken into account, nonlinear fracture mechanics models emerged as a powerful tool for analyzing growth of distributed cracking and its localization in concrete. There is a scattered literature exploring nonlinear fracture behavior of concrete. It consists of numerical and experimental applications of all important nonlinear fracture models to characterize related fracture parameters of concrete. This inspired us to attempt to present a unified view of the theoretical development and applications of these models on concrete.

In the journey of writing this book we came across many legendary researchers who have actively contributed directly or indirectly toward the content of this book.

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Above all, we would like to express profound gratitude and appreciation to Almighty.

Jamshedpur, India Shailendra Kumar
Kharagpur, India Sudhirkumar V. Barai
10 July 2010

Contents

List of Symbols and Abbreviations

List of Symbols

a	crack length
a_{o}	initial crack length
a_{c}	effective crack length at peak (critical) load
a_{e}	effective crack length at peak (critical) load from ECM
B	width of the test specimen
b_{i}	boundary force distribution defined on A_{T}
$[C]$	a vector of crack opening displacement at node i for unit external load
c_1, c_2	material constants for nonlinear softening function
c_{f}	effective extension of crack length for infinitely large specimen
$\{C_{\mathrm{g}}\}$	a vector of crack opening displacement at node i due to self-weight of specimen
$\mathrm{CTOD_{cs}}$	critical value of crack-tip opening displacement from TPFM
c_{p}	length of fully developed fracture process zone
d_{a}	maximum size of aggregates
D	depth of the test specimen (characteristic dimension)
D_{g}	load point deflection due to self-weight of the specimen
D_{L}	load point deflection when external load is unity
$\{D_{\mathrm{p}}\}$	a vector of load point deflection when $\{p\} = \{1\}$
E	modulus of elasticity of concrete
f	a function
F_1, F_2	geometric factors for different loading cases
$F(x/a, a/D)$	the standard Tada Green's function for edge cracks subjected to pair of forces normal to the crack face
f_{c}	cylinder compressive strength
f_{ck}	characteristics strength of concrete
f_{cm}	mean compressive strength
f_{cu}	cube compressive strength of concrete
f_{tm}	mean tensile strength

F_i	the body force
f_t	uniaxial tensile strength of concrete
G_f	initial fracture energy of concrete, defined as the area under curve of the horizontal intercept of the initial tangent of the softening curve
G_F, G_{FC}	fracture energy of concrete, defined as the area under curve of the post-peak stress and crack opening displacement relation
$g_f(x)$	Hillerborg's local cohesive breaking energy at any location x along the FPZ
G_{FB}	fracture energy of concrete for infinitely large specimen
G_{I-coh}	the average value of cohesive breaking energy per unit length
${G_F}^{ini}$	crack initiation fracture energy obtained using LEFM equation from known value of ${K_{IC}}^{ini}$
${G_F}^{un}$	unstable fracture energy obtained using LEFM equation from known value of ${K_{IC}}^{un}$
${G_{IC}}^{ini}$	the crack initiation fracture energy release
${G_{IC}}^{un}$	the unstable fracture energy release
${G_{IC}}^{C}$	the critical value of the cohesive breaking energy
H_o	thickness of the clip gauge holder
$[K]$	a symmetric matrix and the value of K_{ij} is the COD at node i by a unit opening nodal force applied at node j
K_I	stress intensity factor in mode I fracture
${K_I}^{COH}$	the cohesive toughness during crack extension
${K_C}^{t}$	total crack extension resistance
${K_{IC}}^{ini}$	the crack initiation toughness
${K_{IC}}^{un}$	the unstable fracture toughness
${K_{IC}}^{C}$	the cohesive toughness at critical condition
${K_{IC}}^{e}$	critical value of SIF from ECM
${K_{IC}}^{s}$	critical value of SIF from TPFM
${K_{IC}}^{b}$	critical value of SIF from SEM
$\overline{K_{IC}^{ini}}$	the effective crack initiation toughness
$\overline{K_{IC}^{un}}$	the effective unstable fracture toughness
K_{INu}	stress intensity factor for P_u and a_o
k_P	stress intensity factor at the process zone tip due to unit applied load
K_σ	the stress intensity factor at the process zone tip due to cohesive stress
K_{IC}, K_C	the critical stress intensity factor
$[{K_{IC}}^{C}]_{NI}$	${K_{IC}}^{C}$ determined using numerical integration method
$[{K_{IC}}^{ini}]_{NI}$	${K_{IC}}^{ini}$ determined using numerical integration method
$[{K_{IC}}^{C}]_{WF4}$	${K_{IC}}^{C}$ determined using weight function method with four terms

$[K_{IC}{}^{ini}]_{WF4}$	$K_{IC}{}^{ini}$ determined using weight function method with four terms
$[K_{IC}{}^{C}]_{WF5}$	$K_{IC}{}^{C}$ determined using weight function method with five terms
$[K_{IC}{}^{ini}]_{WF5}$	$K_{IC}{}^{ini}$ obtained using weight function method with five terms
$K_{IC}{}^{un*}$	$K_{IC}{}^{un}$ evaluated by Xu and Reinhardt (1999b, c) in Chap. 4
$K_{IC}{}^{ini*}$	the $K_{IC}{}^{ini}$ evaluated by Xu and Reinhardt (1999b, c) in Chap. 4
$k(\alpha)$	a geometric factor
l_{ch}	characteristic length of material
$m(x,a)$	weight function
M_1, M_2, M_3, M_4	parameters of weight function
$\{p\}$	a vector of nodal forces
P_{ini}	initial cracking load
P_u	maximum applied load
$P_{u,exp}$	maximum applied load obtained from experiment
$P_{u,CCM}$	maximum applied load obtained using cohesive crack model
$P_{u,LEFM}$	maximum load obtained using LEFM concept
r_u	undamaged length of the ligament during crack propagation
S	span of the test beam
u_i	an admissible displacement field for the system
W	the strain energy density function defined in the body
w	crack opening displacement ahead of the crack tip
w_1	the horizontal intercept of the initial tangent of softening
w_t	crack opening displacement at the initial notch tip
w_c	maximum COD when the cohesive stress becomes zero
w_g	the self-weight per unit length of the beam
w_s	COD at slope change in the bilinear softening
α	a/D ratio
β_B	the brittleness number
δ	load point deflection
Δa_c	effective crack extension at critical load
ε_{ij}	strain field the surface potential defined in the part of boundary A_P
υ	Poisson's ratio of concrete
$\sigma_s(\mathrm{CTOD_c})$	cohesive stress at $\mathrm{CTOD_c}$
$\sigma(w_t)$	cohesive stress at the initial notch tip
σ	cohesive stress ahead of the crack tip
σ_{Nu}	peak nominal stress
σ_s	cohesive stress at slope change in the bilinear softening
σ_{tip}	stress value generated at the crack tip during loading
Π	the total potential

List of Abbreviations

CBM	crack band model
CCM	cohesive crack model
CMOD	crack mouth opening displacement
$CMOD_c$	critical value of crack mouth opening displacement
COD	crack opening displacement
COD_c	critical value of crack opening displacement
CT	compact tension
CTOD	crack-tip opening displacement
$CTOD_c$	critical value of crack-tip opening displacement
DGFM	double-*G* fracture model
DKFM	double-*K* fracture model
ECM	effective crack model
FCM	fictitious crack model
FEM	finite element method
FPBT	four-point bending test
FPZ	fracture process zone
LEFM	linear elastic fracture mechanics
LHS	left-hand side
RHS	right-hand side
SEM	size-effect model
SIF	stress intensity factor
TPBT	three-point bending test
TPFM	two-parameter fracture model
WST	wedge splitting test

Chapter 1
Introduction to Fracture Mechanics of Concrete

1.1 General

Fracture mechanics is a branch of solid mechanics which deals with the behavior of the material and conditions in the vicinity of a crack and at the crack tip. While the concept of linear elastic fracture mechanics has been well developed for more than past 40 years and successfully applied to metallic structures, several civil engineering materials such as cementitious materials, rocks, and fiber-reinforced composites commonly known as *quasibrittle* need a different fracture mechanics approach to model the fracture process. Cementitious materials can be modeled at various scales like the nano-, micro-, meso-, and macrolevels. At mesolevel, they can be considered as a two-phase particulate composite, i.e., the matrix and the reinforcement. In the cement pastes, mortar, and concrete, the matrices can be considered as the hydrated cement gels, cement paste, and mortar, respectively, whereas the reinforcements in the corresponding materials can be taken as unhydrated cement particles, fine aggregates, and coarse aggregates.

Concrete is made up of many ingredients such as cement, fine aggregates, coarse aggregates, water, and admixtures in complex arrangements. Apart from the two-phase particulate composite, multitudes of internal voids ranging up to several millimeters are present in the hardened concrete (Shah et al. 1995). These voids including pores in cement, cracks at matrix–aggregate interface, and shrinkage cracking have significant influence on the mechanical behavior of concrete. The presence of defects plays an important role and gives rise to a microcracked zone ahead of the tip of a macro-crack, resulting in a progressive material failure as the crack grows. Preceding to unstable or critical load, a sub-critical crack grows when a pre-cracked concrete specimen is loaded. The pre-critical crack growth is often termed as fracture process zone or damage process zone or slow crack growth.

During 1960–1970s, several experimental and numerical investigations proved that the classical form of linear elastic fracture mechanics cannot be applied to normal size concrete members. The inapplicability of linear elastic fracture mechanics was discovered and reasoned as the presence of large and variable size of fracture process zone. From the past research and studies it became clear that the fracture mechanics can be a useful and powerful tool for the analysis of the growth

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of distributed cracking and its localization in concrete if the softening behavior of the material is taken into account. The actual application of tension-softening constitutive law was unknown until about late 1970s. Then using nonlinear fracture mechanics, Hillerborg and co-workers (1976) put forward a pioneer work in which the development of fictitious crack model based on cohesive crack model (Dugdale 1960, Barenblatt 1962) for the crack propagation study of unreinforced concrete beam was introduced. Thereafter, a number of fracture models have been proposed and used to predict the nonlinear fracture behavior of cementitious materials. The other nonlinear models are crack band model (Bažant and Oh 1983), two-parameter fracture model (Jenq and Shah 1985), size-effect model (Bažant 1984, Bažant et al. 1986), effective crack model (Nallathambi and Karihaloo 1986), K_R-curve method based on cohesive force (Xu and Reinhardt 1998, 1999a), double-K fracture model (Xu and Reinhardt 1999a), and double-G fracture model (Xu and Zhang 2008). More appropriately, the cohesive crack and crack band models are attributed to numerical approach such as finite element or boundary element technique, whereas the others models employ the concept of linear elastic fracture mechanics in its modified form.

Design of concrete structures has already passed through two important phases in its historical evolution. The first phase of development took place until 1930s with the elastic no-tension analysis and in its second phase during 1940–1970, the plastic limit theory was introduced. Since concrete structures have been designed and successfully built according to national codes of practice without using the concept of fracture mechanics, it might seem unnecessary to change the current design practice (ACI Committee-446 1992, Bažant and Planas 1998). However, a reasonable consensus has appeared from the researchers and professionals around the world to explore the possibility of introducing the fracture mechanics into the design practices. There are good reasons to believe that the third phase of evolution in the design of concrete structures may probably come by introduction of fracture mechanics parameters. The most important reasons (ACI Committee-446 1992, Karihaloo 1995, Bažant and Planas 1998) for the need of evoking fracture mechanics are briefly mentioned as below:

- Lack of energy criterion for the crack development and its growth in the existing national design codes.
- Application of fracture mechanics will avoid the unobjectivity in the results.
- Due to lack of yield plateau and presence of material softening damage or cracking behavior, the plastic hinges do not form at isolated locations. Instead, the failure is associated with the propagation of the failure zone throughout the structure.
- The existing failure theory cannot give information on the post-peak response in order to obtain the energy absorption under the complete load–deflection curve. This information can be conveniently determined using fracture mechanics theory.
- The concrete structures exhibit size effect due to various reasons including boundary layer or wall effect, diffusion phenomena, hydration heat, statistical

size effect, fracture mechanics size effect, and fractal nature of crack surfaces (Bažant and Planas 1998). Out of these, the most important size effect is due to the release of stored energy of the bulk of the structure into the fracture surface energy (fracture mechanics size effect). This particular behavior can be easily modeled using the fracture mechanics principle.

- Besides above, the brittleness of the material can be obtained quantitatively with the help of fracture mechanics concept. This parameter may be useful for measuring the ductility property of concrete, especially for high-strength concrete, which shows relatively more brittle failure. Moreover, more uniform safety of factors can be achieved in the structural design.

The preceding investigations show that the nonlinear fracture behavior is significantly influenced by softening function of concrete. Many shapes of softening curve have been proposed by different groups of researchers. In real sense, the actual shape of the softening can be determined from a direct tension test on the concrete specimen. However, indirect methods using notched specimens have been also proposed in the past to avoid the intricacy involved in the true (direct) tensile tests. In the indirect method, the load–displacement curves obtained from the experiment and theoretical prediction are compared until they are up to a limited error tolerance. Trial-and-error or optimization method is adopted to gain an accurate softening curve until the theoretically predicted and experimentally observed load–displacement curves are in a good agreement within the acceptable error range.

For practical engineering applications of the fracture mechanics-based concept to the design of concrete structures, some kind of simplicity in the structural analysis is always welcomed. The nonlinear fracture models based on the numerical approach are relatively more involved in the computations. For this reason, probably, the fracture models based on the modified linear elastic fracture mechanics may bridge the gap between the computational efficiency and the model predictive capability of results, because they are relatively more computationally efficient but have limited capacity to predict the fracture parameters. Nevertheless, recently proposed double-*K* fracture model, K_R-curve method based on cohesive stress distribution in fracture process zone and the double-*G* fracture model belonging to the modified linear elastic fracture mechanics concept can predict more number of fracture parameters without needing closed-loop testing system in the experiments. Precisely, they can predict the important stages of fracture processes like crack initiation, stable crack propagation, and unstable fracture. Moreover, with the use of K_R-curve method, the complete fracture process can be analyzed. However, calculation method of the fracture models, particularly double-*K* fracture model and K_R-curve method, needs specialized numerical technique because of singularity problem at the integral boundary. To avoid this, a simplified method (Xu and Reinhardt 2000) has been proposed in the past which uses two empirical relationships for determining the double-*K* fracture parameters.

One of the major reasons which possibly inhibit the wide application of the fracture parameters to a common practice is that the concrete fracture parameters are influenced by many factors including softening function of concrete, concrete strength, specimen size, specimen geometry, geometrical factor like relative size of notch length and the loading condition.

The literature reports extensive numerical and experimental investigation on nonlinear fracture behavior of concrete. All the important nonlinear fracture models are widely applied to characterize the related fracture parameters. However, a systematic study using cohesive crack model with important softening functions of concrete may be a useful supplement. Further, closed-form equation solutions for determining the double-K fracture parameters and the K_R curve associated with cohesive stress in the fictitious fracture zone will lead in computational efficiency and avoid the need of the specialized numerical technique. From the observations based on the experimental studies (Xu and Reinhardt 1999a, b), it was reported that the double-K fracture parameters are almost independent of the specimen size. Though the size-effect study of double-K fracture parameters based on experimental results is available in the literature, no such numerical study has been attempted in the past. The relationship between double-K and double-G fracture parameters using experimental tests has been demonstrated, the same using numerically obtained data has not been presented in the literature. The various studies including influence of specimen geometry and the type of loading condition on the double-K fracture parameters and the K_R curves are not available in the literature. It is interesting to carry out those studies. The load–displacement curves are the preliminary requirements for the above studies. For the investigation of the influence of specimen geometry, the results of load–displacement curves tested under the similar conditions for different specimen geometries are needed. Similarly, for the effect of loading condition, the load–displacement plots tested under the similar conditions except for different loading conditions are required. Most of the experimentally measured load–displacement curves available in the literature are not favorable to investigate the influence of specimen geometry and loading condition on the above fracture parameters. In other way, these investigations should be carried out with the load–displacement curves generated using nonlinear fracture models like the cohesive crack model or the crack band model. The advantage of gaining such load–displacement curves is that they may be free from unseen errors due to experimental measurements. The above lacunas were considered in the doctoral research program of Kumar (2010), who presented some of the behavior of fracture parameters based on numerical studies. In this text book, most of the contents are taken from the works of Kumar (2010) and Kumar and Barai (2008a–c, 2009a–f, 2010a, b) but more in an elaborative manner. In the subsequent chapters, cohesive crack model for three-point bending test and compact tension specimens using important softening functions of concrete are developed. The numerical results are compared with the experimental results available in the literature. Further, a systematic study on the different fracture parameters is carried out. Weight function method is introduced to determine the double-K and the K_R-curve method based on cohesive stress distribution. Application of universal form of weight function enables one to obtain closed-form equation for the cohesive toughness of the material during crack extension. The validity of the formulations is established by comparing results obtained from existing analytical or simplified methods using both the experimental and numerical input data. Furthermore, attempts are made to put forward a possible new equation of brittleness of concrete with the help of double-K fracture parameters and formulize the size-effect predictions from the double-K fracture using the cohesive crack model as the reference. A comprehensive comparison between the double-K and double-G fracture criteria is presented. Emphasis on the effect of various parameters including specimen geometry, size effect, and loading condition on the double-K and the K_R-curve method is also focused.

1.2 Organization of the Book

The textbook presents results of a comprehensive study on the crack initiation and its growth in concrete-like materials using various fracture models. A brief outline of the book consisting of six chapters as shown in Fig. 1.1 is given below:

- Chapter 1 presents the general introduction of the book briefly.
- Chapter 2 gives a state-of-the-art review on various aspects of the material behavior and development of different concrete fracture models. The critical observations on the available literature and the scope of the present book are also highlighted.
- Chapter 3 presents the development of cohesive crack or fictitious crack model for two standard tests: (i) three-point bending test and (ii) compact test specimens using commonly used softening functions of concrete. The results are analyzed and compared with the experimental results available in literature. The size-effect study between the size-effect model and the cohesive crack model is also carried out.
- Chapter 4 contains the extensive study on the double-K and double-G fracture parameters. Initially, the weight function approach is introduced for determining the double-K fracture parameters. This approach is compared with the existing *analytical method* and *simplified approach*. Prediction of size effect from the double-K fracture parameters is formulized. Study of the influence of many factors including specimen geometry, loading condition, size effect, and softening function on the double-K fracture parameters is presented. At last, a comparative study between double-K and double-G fracture parameters is carried out.
- Chapter 5 presents the application of weight function approach for determining the K_{R} curve associated with cohesive stress distribution in the fracture zone. This method is validated with the existing *analytical method* using test data available in the literature. Further, effect of specimen geometry, loading condition, size effect, and softening function on the K_{R} curve is investigated.
- Chapter 6 presents a comparative study between various fracture parameters obtained from cohesive crack model, two-parameter fracture model, size-effect model, effective crack model, double-K fracture model, and double-G fracture model. Finally this chapter ends with final closing remarks.

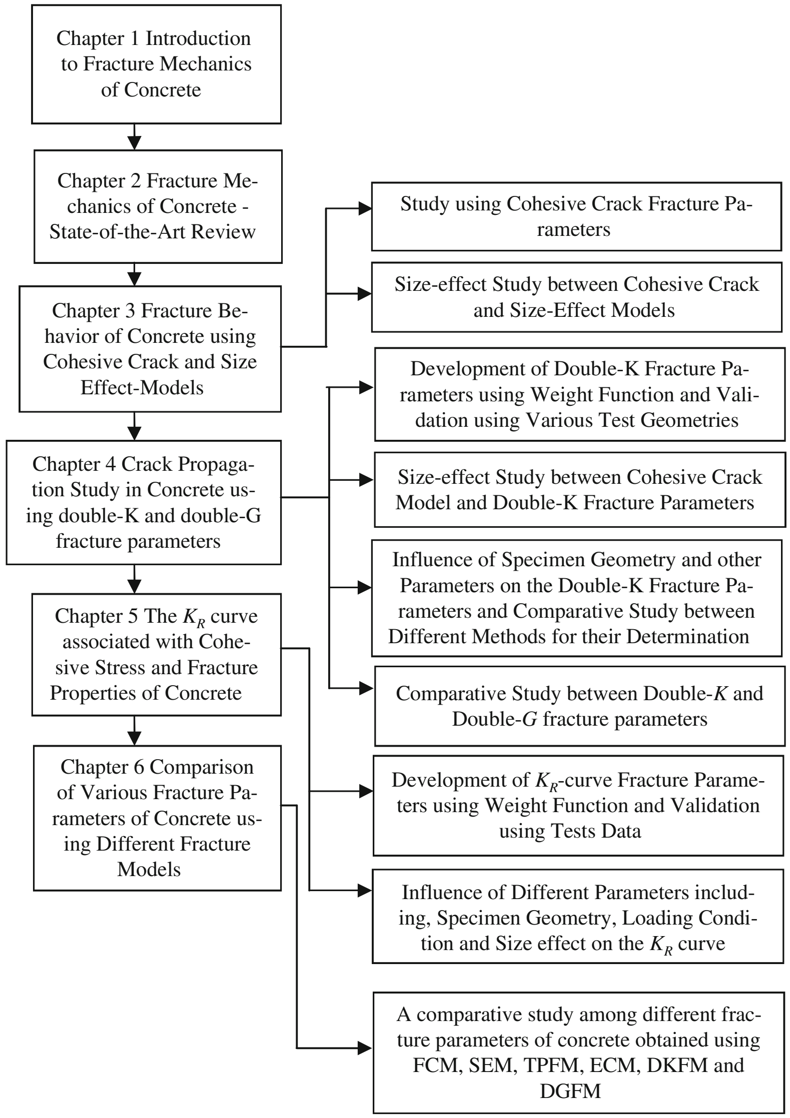

Fig. 1.1 Outline of the book

1.3 Closing Remarks

A brief application of the fracture mechanics concepts to concrete was addressed in this chapter. The important reasons why fracture mechanics to concrete can be applied were focused. At the end, organization of the book was addressed.

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Chapter 2
Fracture Mechanics of Concrete – State-of-the-Art Review

2.1 Introduction

The genesis of the development of fracture mechanics goes back to the beginning of 20th century when Inglis (1913) published a pioneer work on stress analysis for an elliptical hole in an infinite linear elastic plate loaded at its outer boundaries, in which a crack-like discontinuity was modeled and stress singularity was observed at the crack tip by making the minor axis very much less than the major axis. The actual development in this field could not occur until a new approach was postulated by Griffith (1921). Since then, it took around another four decades when for the first time the concept of fracture mechanics was applied to cementitious materials. Thereafter, the study of crack propagation in cement-based materials and structures attracted interest of a large number of researchers around the world until today. In this chapter, a state-of-the-art review on various aspects of fracture process of concrete-like materials now-a-days called as *quasibrittle* materials is presented.

2.2 Linear Elastic Fracture Mechanics

From analysis of a sharp crack in a sheet of brittle material (glass) subjected to a constant remotely applied stress, Griffith (1921) presented the first explanation of the mechanism of brittle fracture using a new energy-based failure criterion. According to this criterion, a certain amount of the accumulated potential energy in the system must decrease to overcome the surface energy of the material in order for the crack to propagate. It was shown that the stresses near the crack tip tend to approach infinity.

The Griffith's theory for ideally brittle materials was extended to account for the limited plasticity near the crack tip in majority of the engineering materials (Irwin 1955, Orowan 1955). It was postulated that the resistance to crack extension is taken as sum of the elastic surface energy and the plastic work. For ductile materials, the plastic dissipation energy is much greater than the elastic energy; therefore, resistance to crack growth is mainly governed due to plastic work.

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Later, based on the existing mathematical procedures (Westergaard 1939), a series of linear elastic crack stress fields was developed (Irwin 1957) and it was shown that the stress field near a sharp crack tip shows a fundamental singular variation, i.e., the stress near the crack tip decreases in proportion to the square root of the distance to the crack tip r ($r << a$, and a is the crack length). In a cracked body, the singularity is independent of the boundary conditions, geometry, loading, and the type of cracking mode for which three fundamentally different types of failure modes such as mode I, mode II, and mode III as shown in Fig. 2.1 exist. For mode I cracking, the stresses and displacements near the crack tip as shown in Fig. 2.2 and expressed by Eqs. (2.1) and (2.2) are given in polar coordinates with origin at the crack tip, at a distance r and angle θ:

$$\begin{aligned}\sigma_{xx} &= \frac{K_\mathrm{I}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\\ \sigma_{yy} &= \frac{K_\mathrm{I}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)\\ \tau_{xy} &= \frac{K_\mathrm{I}}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}\end{aligned} \tag{2.1}$$

$$\begin{aligned}u &= \frac{K_\mathrm{I}\,(1+\upsilon)}{E}\sqrt{\frac{2r}{\pi}}\cos\frac{\theta}{2}\left(\frac{k-1}{2}+\sin^2\frac{\theta}{2}\right)\\ v &= \frac{K_\mathrm{I}\,(1+\upsilon)}{E}\sqrt{\frac{2r}{\pi}}\sin\frac{\theta}{2}\left(\frac{k-1}{2}-\cos^2\frac{\theta}{2}\right)\end{aligned} \tag{2.2}$$

The parameter k is $(3-\upsilon)/(1+\upsilon)$ and $(3–4\upsilon)$ in plane stress and plane strain conditions, respectively. The value K_I is called as the stress intensity factor (SIF) for mode I situation. Similar expressions for the stresses and displacements in the mode II and mode III failure conditions can be written in which the stress intensity factors are denoted by K_II and K_III, respectively.

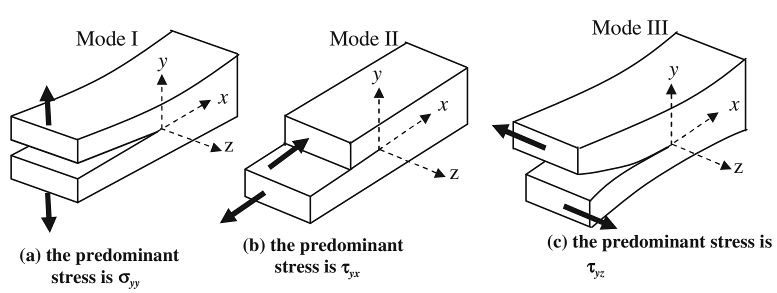


Fig. 2.1 Cracking modes: (**a**) mode I (or opening mode), (**b**) mode II (or sliding or in-plane shear mode), (**c**) mode III (or the tearing or anti-plane shear mode)

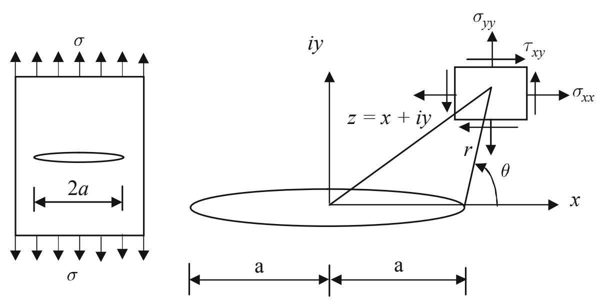


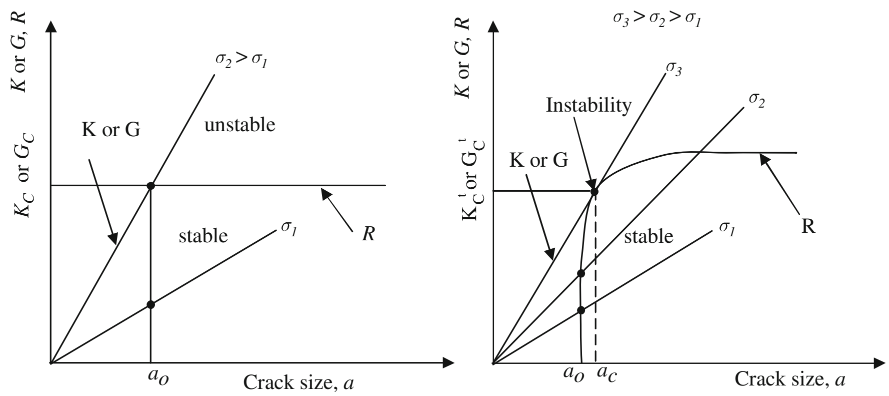
Fig. 2.2 Stress field in the vicinity of crack tip for mode I fracture using Westergaard complex coordinates

2.2.1 Significance of Stress Intensity Factor

The term stress intensity factor is different from the term stress concentration factor, which is used to characterize the ratio between actual and average or nominal stress at a geometric discontinuity. On the other hand, the stress intensity factor defines the amplitude of the crack-tip singularity, that is, the stress field in the vicinity of the crack tip increases proportionally to stress intensity factor. When a structural component is stressed in tension or bending, the developed stress field in the vicinity of a crack tip under elastic conditions shows a singularity following an inverse square root relationship with distance from the crack tip. In other words, stress intensity factor describes the strength of this singularity. Since the stress and displacement field in the vicinity of the crack tip is controlled by the stress intensity factor, it may be further assumed that critical stress or displacement condition at the crack tip can be explained using a critical value of stress intensity factor for any modes of failure. Hence the concepts of linear elastic fracture mechanics may be reasonably characterized using a single parameter, that is, stress intensity factor. Furthermore, local yielding occurs in engineering materials that relieves the singularity and hence the size of the plastic zone can be directly related to the stress intensity factor.

2.2.2 Concept of R Curve

In early 1960s, the *R*-curve approach (Irwin 1960, Krafft et al. 1961) based on energy balance was proposed. In the *R*-curve concept, an energy release rate *G* as a measure of the energy available for an increment of crack extension was postulated. According to definition, crack extension occurs when strain energy release rate *G* is equal to *R*, where *R* is called the material resistance to crack extension. The *R* curve is represented by a plot between the crack extension resistance expressed in terms of either strain energy release rate *G* or stress intensity factor *K* and the corresponding crack extension Δa as shown in Fig. 2.3. It is generally called as G_R curve and K_R curve depending upon the unit of the crack resistance parameters strain energy



(a) Flat *R* curve **(b) Rising *R* curve**

Fig. 2.3 Representation of *R* curve

release rate and stress intensity factor, respectively. The shape of the *R* curve determines the onset of crack instability. For most of the engineering materials except truly brittle material, a rising *R* curve is observed due to slow stable crack extension. As one crack extends, a rising *R* curve produces several small cracks because the required stress intensity to propagate the crack will increase to a point where it is favorable to propagate a different crack. On the other hand, a flat *R* curve for ideal brittle materials will give rise to a single and possibly catastrophic crack. Depending on how *G* and *R* vary with the crack size, the crack growth may be stable or unstable.

In Fig. 2.3a for a flat *R* curve, the crack is stable between the stress levels σ_1 and σ_2 and beyond the stress σ_2, the crack propagates unstably. In this case the total value of critical energy release is G_C. In Fig. 2.3b for a rising *R* curve, the crack extension is stable between the stress levels σ_2 and σ_3 and beyond the stress σ_3, the crack propagates unstably. At the onset of unstable crack growth, the driving force is the tangent to the *R* curve and corresponding value of the total crack extension resistance at the point of tangency between *G* and *R* curves is $G_C{}^t$ at the critical effective crack extension a_c.

The fracture criterion and instability of a propagating crack in cracked solid structures represented using the K_R curve can be expressed mathematically using Eq. (2.3):

$$\frac{\partial K}{\partial a} < \frac{\partial K_R}{\partial a} \text{ The crack extension takes place in stable manner}$$

$$\frac{\partial K}{\partial a} = \frac{\partial K_R}{\partial a} \text{ Critical condition is reached at onset of unstable crack extension}$$

$$\frac{\partial K}{\partial a} \geq \frac{\partial K_R}{\partial a} \text{ The crack propagates uncontrollably} \tag{2.3}$$

2.3 Elastic–Plastic Fracture Mechanics

Linear elastic fracture mechanics (LEFM) is applicable to the materials in which the nonlinear behavior is confined to a small region near the crack tip. There are many materials, however, for which the applicability of LEFM is impossible or at least suspicious. Moreover, the *R*-curve concept describes the fracture behavior of a material of limited ductility in the vicinity of a crack tip for plane stress situation. Therefore, an alternative elastic–plastic fracture mechanics is applied for too ductile material behavior which exhibits time-independent, nonlinear behavior (plastic deformation). In the nonlinear behavior at the crack tip, the fracture criterion is characterized by two parameters:

1. crack-tip opening displacement (CTOD) criterion (Wells 1962) and
2. the *J*- integral approach (Rice 1968a, b).

Critical values of CTOD and *J*-integral give nearly size-independent measures of fracture toughness, even for relatively large amount of crack-tip plasticity. However, there are still limits to the applicability of *J*-integral and CTOD criteria, but these limits are much less restrictive than the validity requirements of LEFM (Anderson 2005).

2.3.1 The CTOD Criterion

The stability analysis for crack growth in the CTOD criterion is based on the characteristic critical value of crack-tip opening displacement CTODc. According to this approach the crack is stable as long as CTOD $\leq$ $\mathrm{CTOD_c}$ (Wells 1962).

2.3.2 The J-Integral Approach

A path-independent contour integral called *J*-integral was introduced (Rice 1968a, b) for the analysis of crack extension. The value of *J*-integral is equal to the energy release rate in a nonlinear elastic cracked body and can be viewed as both an energy parameter and a stress intensity parameter. Considering an arbitrary counterclockwise path Γ as shown in Fig. 2.4 around the crack tip, the path-independent *J*-integral is given by

$$J = \int_{\Gamma} \left(W\,\mathrm{d}y - T_i \frac{\partial u_i}{\partial x} \mathrm{d}s \right) \tag{2.4}$$

where W is the strain energy density $= \int_0^{\varepsilon_{ij}} \sigma_{ij}\,\mathrm{d}\varepsilon_{ij}$; T_i is the component of the traction vector $= \sigma_{ij} n_j$, being the components of the unit vector normal to Γ;

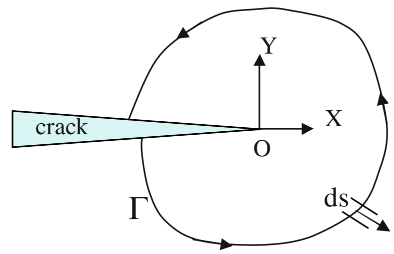


Fig. 2.4 The definition of the *J*-integral

u_i is the displacement vector components; ds is the length increment along the contour Γ. The characteristic critical value of the *J*-integral J_c is considered to define the following form of the crack initiation criterion:

$$J = J_c \tag{2.5}$$

where J_c is considered as a material fracture parameter.

2.4 Early Research Using LEFM to Concrete

The application of LEFM concept to concrete was first attempted in early 1970s by Kaplan (1961). Notched concrete beams tested in three-point bending and four-point bending configurations were used to measure the critical strain energy release rate of concrete. The results indicate that the critical strain energy release rate of concrete depends on the concrete mix proportion, type of loading condition, relative size of initial notch length, and specimen dimension. It was pointed out that the proper account of slow crack growth prior to fast fracture could be introduced in the process of fracture analysis. However, the Griffith concept of a critical energy rate being a condition for rapid crack propagation may be applicable to concrete after suitable modification in analytical and experimental procedures. Thereafter, many experimental and numerical investigations have been performed to predict fracture behavior of concrete. Researchers in the 1960s used LEFM to cementitious materials similar to the experimental determination of critical stress intensity factor K_{IC} in metals.

Glucklich (1963) examined the fracture of concrete using fracture mechanics approach and revealed that the strain energy is converted mainly to surface energy but the surface involved is much larger in area than the surface of the effective crack. The rate of energy absorption at the crack tip in concrete is mainly due to entire highly stressed zone in the form of microcracking and not to plastic flow. The increase of the microcracked zone and the heterogeneity of the composite materials contribute to the relatively high value of the strain energy release rate. The driving force (strain energy release rate) increases with the crack length in tension, whereas it is a constant value in compression fracture.

Naus and Lott (1969) conducted experimental investigation to determine the effects of several concrete parameters: water–cement ratio, air content, sand–cement ratio, curing age, and size and type of coarse aggregate on the effective fracture toughness of concrete using three-point bending test. A regular dependency of the effective fracture toughness was observed with the variation of concrete parameters.

Shah and McGarry (1971) concluded that hardened Portland cement paste is a notch-sensitive material, whereas mortar and concrete with the normally used amounts and volume of stone aggregates are notch-insensitive materials with notch length lower than at least a few centimeters. The critical notch length for mortar and concrete depends on the volume, type, and size of aggregate particles.

Walsh (1971) presented test results of fracture toughness of geometrically similar notched concrete beams tested in three-point bending geometry. The test results of nominal strength and their sizes were plotted on double logarithmic scale. From the graphs, it was found that the results did not follow the straight line of slope –1/2, which inferred that the LEFM was not applicable to concrete. He suggested that the crack propagation load obtained using LEFM depends upon the size of the specimen. Further, in order to apply the LEFM to concrete, a minimum value of ligament length 150 mm for a beam depth 225 mm should be used for fracture testing.

Brown (1972) used two methods: a notched-beam technique and a double-cantilever beam to measure the effective fracture toughness of cement pastes and mortars. Tests of both pastes and mortars showed that the fracture toughness of cement is independent of crack growth but that the toughness of mortar increases as the crack propagates.

Kesler et al. (1972) performed the experimental investigation on a large number of cracked cement paste, mortar, and concrete specimens with an objective to determine the applicability of the LEFM to these materials. Based on the analysis of the test results, the authors concluded that the concepts of LEFM cannot be directly applicable to the cementitious materials having sharp cracks.

Brown and Pomeroy (1973) determined the effective fracture toughness on cement paste and mortar using both a notched-beam and a double-cantilever beam method. The influence of size and quality of aggregates on the effective fracture toughness was reported in the experimental research. It was found that the addition of aggregate not only increases the toughness but also results in a progressive increase in toughness with crack growth. The higher the proportion of aggregate, the larger the increase in toughness: fine aggregate is possibly more effective than coarse in this respect.

Fracture mechanics studies on plain and polymer-impregnated mortars (Evans et al. 1976) showed that macro-crack propagation resistance of these materials is not significantly influenced by water/cement ratio and curing time but is greatly enhanced by polymer impregnation. The fracture parameters are independent of the crack length for cracks larger than ~2 cm. Acoustic emission measurements showed that the susceptibility to microcracking is substantially restarted by polymer impregnation.