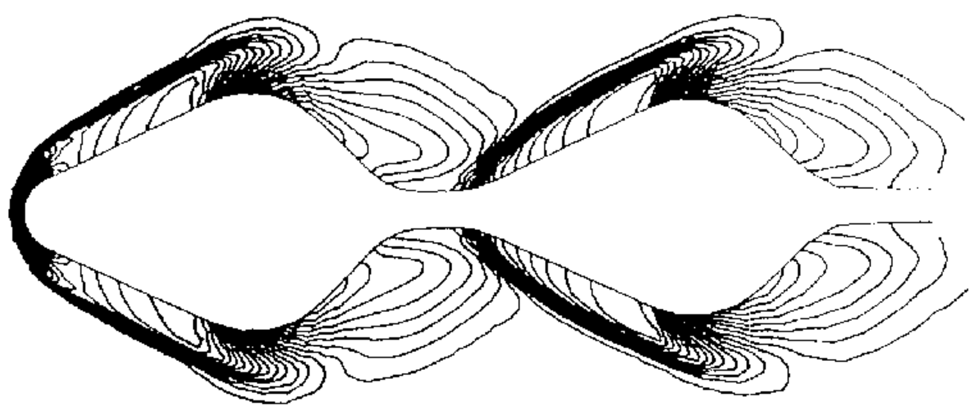
COMPUTATIONAL FLUID DYNAMICS
VOLUME II FOURTH EDITION

Klaus A. Hoffmann
Steve T. Chiang



Fourth Edition

COMPUTATIONAL FLUID DYNAMICS
VOLUME II

KLAUS A. HOFFMANN

STEVE T. CHIANG

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PREFACE

The fundamental concepts of computational schemes established in the first volume are extended to the solution of Euler equations, Parabolized Navier-Stokes equations, and Navier-Stokes equations, along with treatment of boundary conditions. In addition, chemically reacting flows, unstructured grids, finite volume schemes, and finite element method at the introductory level are included.

This volume begins with a review of the basic concepts which is presented in Chapter 10. Subsequently, the transformation of the equations of fluid motion from physical space to computational space is provided in Chapter 11. This chapter also includes the linearization of the equations as well as the derivation of the Jacobian matrices. Chapter 12 presents numerical schemes for the solution of the Euler equations for inviscid flowfields. Specifications of the boundary conditions, along with illustrated examples, are provided in this chapter. Chapter 13 presents Parabolized Navier-Stokes (PNS) equations and a numerical algorithm for solution. The shock fitting procedure is discussed in this chapter as well. The Navier-Stokes equations and various numerical schemes for solutions are discussed in Chapter 14. Specification of boundary conditions, derivation of governing equations, and comparison of several types of boundary conditions are provided in Chapter 15. An extension of the governing equations to include the effect of chemistry for hypersonic flowfield computations is included in Chapter 16. To familiarize the reader with unstructured grids which are used in conjunction with finite volume and finite element schemes, they are introduced in Chapter 17. It develops some fundamental concepts and explores two techniques for generation of unstructured grids in two-dimensions. Finally, finite volume schemes and finite element method are developed at the introductory level in Chapters 18 and 19, respectively.

Several computer codes are developed based on the materials presented in this text. These codes, manuals, and additional examples are presented in the text, *Student Guide to CFD–Volume II.*

Finally, our sincere thanks and appreciation are extended to all individuals acknowledged in the preface of the first volume. Thank you all very much for your friendship and encouragement.

Klaus A. Hoffmann
Steve T. Chiang

Chapter 10

A Review

10.1 Introductory Remarks

The fundamental concepts of computational fluid dynamics were introduced in the previous chapters. Various aspects of numerical schemes were explored with regard to simple partial differential equations. In all cases up to Chapter 8, the investigations were limited to a single equation. In the upcoming chapters the concepts are extended to systems of equations. Before proceeding further, however, it is beneficial to review and summarize the content of the previous chapters.

10.2 Classification of Partial Differential Equations

Partial differential equations (PDEs) can be classified into different categories, where within each category they may be classified further into subcategories. The numerical procedure used to solve a partial differential equation very much depends on the classification of the governing equation. A brief review of the classification of partial differential equations is provided in the following subsections.

10.2.1 Linear and Nonlinear PDEs

(a) Linear PDE: There is no product of the dependent variable and/or product of its derivatives within the equation.

(b) Nonlinear PDE: The equation contains a product of the dependent variable and/or a product of the derivatives.