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EXTENDED FINITE ELEMENT METHOD

XFEM

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EXTENDED FINITE ELEMENT METHOD

for Fracture Analysis of Structures

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To Mansoureh

Preface

'*I am always obliged to a person who has taught me a single word.*'

Progressive failure/fracture analysis of structures has been an active research topic for the past two decades. Historically, it has been addressed either within the framework of continuum computational plasticity and damage mechanics, or the discontinuous approach of fracture mechanics. The present form of linear elastic fracture mechanics (LEFM), with its roots a century old has since been successfully applied to various classical crack and defect problems. Nevertheless, it remains relatively limited to simple geometries and loading conditions, unless coupled with a powerful numerical tool such as the finite element method and meshless approaches.

The finite element method (FEM) has undoubtedly become the most popular and powerful analytical tool for studying a wide range of engineering and physical problems. Several general purpose finite element codes are now available and concepts of FEM are usually offered by all engineering departments in the form of postgraduate and even undergraduate courses. Singular elements, adaptive finite element procedures, and combined finite/discrete element methodologies have substantially contributed to the development and accuracy of fracture analysis of structures. Despite all achievements, the continuum basis of FEM remained a source of relative disadvantage for discontinuous fracture mechanics. After a few decades, a major breakthrough seems to have been made by the fundamental idea of partition of unity and in the form of the eXtended Finite Element Method (XFEM).

This book has been prepared primarily to introduce the concepts of the newly developed extended finite element method for fracture analysis of structures. An attempt has also been made to discuss the essential features of XFEM for other related engineering applications. The book can be divided into four parts. The first part is dedicated to the basic concepts and fundamental formulations of fracture mechanics. It covers discussions on classical problems of LEFM and their extension to elastoplastic fracture mechanics (EPFM). Issues related to the standard finite element modelling of fracture mechanics and the basics of popular singular finite elements are reviewed briefly.

The second part, which constitutes most of the book, is devoted to a detailed discussion on various aspects of XFEM. It begins by discussing fundamentals of partition of unity and basics of XFEM formulation in Chapter 3. Effects of various enrichment functions, such as crack tip, Heaviside and weak discontinuity enrichment functions are also investigated. Two commonly used level set and fast marching methods for tracking moving boundaries are explained before the chapter is concluded by examining a number of classical problems of fracture mechanics. The next chapter deals with the orthotropic fracture mechanics as an extension of XFEM for ever growing applications of composite materials. A different set of enrichment functions for orthotropic media is presented, followed by a number of simulations of benchmark orthotropic problems. Chapter 5, devoted to simulation of cohesive cracks by XFEM, provides theoretical bases for cohesive crack models in fracture mechanics, classical FEM and XFEM. The snap-back response and the concept of critical crack path are studied by solving a number of classical cohesive crack problems.

The third part of the book (Chapter 6) provides basic information on new frontiers of application of XFEM. It begins with discussions on interface cracking, which include classical solutions from fracture mechanics and XFEM approximation. Application of XFEM for solving contact problems is explained and numerical issues are addressed. The important subject of dynamic fracture is then discussed by introducing classical formulations of fracture mechanics and the recently developed idea of time–space discretization by XFEM. New extensions of XFEM for very complex applications of multiscale and multiphase problems are explained briefly.

The final chapter explains a number of simple instructions, step-by-step procedures and algorithms for implementing an efficient XFEM. These simple guidelines, in combination with freely available XFEM source codes, can be used to further advance the existing XFEM capabilities.

This book is the result of an infinite number of brilliant research works in the field of computational mechanics for many years all over the world. I have tried to appropriately acknowledge the achievements of corresponding authors within the text, relevant figures, tables and formulae. I am much indebted to their outstanding research works and any unintentional shortcoming in sufficiently acknowledging them is sincerely regretted. Perhaps such a title should have become available earlier by one of the pioneers of the method, i.e. Professor T. Belytschko, a shining star in the universe of computational mechanics, Dr J. Dolbow, Dr N. Moës, Dr N. Sukumar and possibly others who introduced, contributed and developed most of the techniques.

I would like to extend my acknowledgement to Blackwell Publishing Limited, for facilitating the publication of the first book on XFEM; in particular N. Warnock-Smith, J. Burden, L. Alexander, A. Cohen and A. Hallam for helping me throughout the work. Also, I would like to express my sincere gratitude to my long-time friend, Professor A.R. Khoei, with whom I have had many discussions on various subjects of computational mechanics, including XFEM. Also my special thanks go to my students: Mr A. Asadpoure, to whom I owe most of Chapter 4, Mr S.H. Ebrahimi for solving isotropic examples in Chapter 3 and Mr A. Forghani for providing some of the results in Chapter 5.

This book has been completed on the eve of the new Persian year; a ‘temporal interface’ between winter and spring, and an indication of the beginning of a blooming season for XFEM, I hope.

Finally, I would like to express my gratitude to my family for their love, understanding and never-ending support. I have spent many hours on writing this book; hours that could have been devoted to my wife and little Sogol: the spring flowers that inspire the life.

Soheil Mohammadi
Tehran, Iran
Spring 2007

Nomenclature

α	Curvilinear coordinate
α_c	Load factor for cohesion
α_f, α_s	Thermal diffusivity of fluid and solid phases
β	Curvilinear coordinate
γ_s	Surface energy density
$\gamma_s^\mathrm{e}, \gamma_s^\mathrm{p}$	Elastic and plastic surface energies
γ_{xy}	Engineering shear strain
δ	Plastic crack tip zone
δ	Variation of a function
$\delta(\xi)$	Dirac delta function
δ_{ij}	Kronecker delta function
$\boldsymbol{\varepsilon}$	Strain tensor
$\boldsymbol{\varepsilon}_f, \boldsymbol{\varepsilon}_c$	Strain field at fine and coarse scales
ε_{ij}	Strain components
$\bar{\varepsilon}_{ij}$	Dimensionless angular geometric function
$\varepsilon_{ij}^\mathrm{aux}$	Auxiliary strain components
ε_v	Kinetic mobility coefficient
ε_yld	Yield strain
η	Local curvilinear (mapping) coordinate system
θ	Crack propagation angle with respect to initial crack
θ	Angular polar coordinate
κ, κ'	Material parameters
λ	Lame modulus
λ	Eigenvalue of the characteristic equation
μ	Shear modulus
ν, ν_{ij}	Isotropic and orthotropic Poisson's ratios
ξ	Local curvilinear (mapping) coordinate system
$\xi(\mathbf{x})$	Distance function
ρ	Radius of curvature
ρ	Density
ρ_f, ρ_c	Density of fine and coarse scales
ρ_int	Curvature of the propagating interface
$\boldsymbol{\sigma}$	Stress tensor
$\boldsymbol{\sigma}_f, \boldsymbol{\sigma}_c$	Stress field at fine and coarse scales
$\boldsymbol{\sigma}_g$	Stress tensor at a Gauss point
$\boldsymbol{\sigma}_t^\mathrm{tip}$	Normal tensile stress at crack tip
σ_0	Applied normal traction

σ_{cr}	Critical stress for cracking
σ_{ij}	Stress components
$\bar{\sigma}_{ij}$	Dimensionless angular geometric function
σ_{ij}^{aux}	Auxiliary stress components
σ_{n}	Stress component normal to an interface
σ_{n}	Stress component at time step n
σ_{yld}	Yield stress
$\boldsymbol{\tau}$	Deviatoric stress
τ_0	Applied tangential traction
τ_{c}	Cohesive shear traction
τ_{n}	Time functions
τ_n	Deviatoric stress tensor at time step n
$\phi(\mathbf{x})$	Level set function
$\phi(z)$	Complex stress function
$\phi_{\text{s}}(z)$	Stress function for shear problem
φ	Angle of orthotropic axes
φ	Phase angle for interface fracture
$\chi(\mathbf{x})$	Enrichment function for weak discontinuities
$\chi(z)$	Stress function
$\psi(\mathbf{x})$	Enrichment function
$\psi(z)$	Stress function
ω	Oscillation index
Γ	Boundary
Γ_{c}	Crack boundary
Γ_{t}	Traction (natural) boundary
Γ_{u}	Displacement (essential) boundary
Δ	Finite variation of a function
Λ	Coefficient matrix
$\boldsymbol{\Xi}$	Homogenisation/average operator
Π	Potential energy
$\Phi_j(\mathbf{x})$	Moving least squares shape functions
$\Phi(\mathbf{x})$	Stress function
Ω	Domain
Ω_f, Ω_c	Fine and course scale domains
Ω_f, Ω_s	Fluid and solid domains
Ω_{pu}	Domain associated with the partition of unity
a	Crack length/half length
a	Semi-major axis of ellipse
$a^{\text{b}}, a^{\text{f}}$	Backward and forward indexes in fast marching method
$\mathbf{a}_h$	Heaviside enrichment degrees of freedom
$\mathbf{a}_i$	Enrichment degrees of freedom
$\mathbf{a}_k$	Enrichment degrees of freedom
A^*	Area associated with the domain J integral
b	Width of a plate
b	Semi-minor axis of ellipse
$\mathbf{b}_i$	Crack tip enrichment degrees of freedom

$\mathbf{B}$	Matrix of derivatives of shape functions
$\mathbf{B}^h$	Matrix of derivatives of final shape functions
$\mathbf{B}_c$	**B** matrix for coarse scale
$\mathbf{B}_f$	**B** matrix for fine scale
$\mathbf{B}_i^r$	Strain–displacement matrix (derivatives of shape functions)
$\mathbf{B}_i^{\mathbf{u}}$	Strain–displacement matrix (derivatives of shape functions)
$\mathbf{B}_i^{\mathbf{a}}$	Matrix of derivatives of enrichment (Heaviside) of shape functions
$\mathbf{B}_i^{\mathbf{b}}$	Matrix of derivatives of enrichment (crack tip) of shape functions
c	Constant parameter
c	Size of crack tip contour for J integral
c_{ij}	Material constants
c_{R}	Rayleigh speed
c_f, c_s	Specific heat for fluid and solid phases
C	Material constitutive matrix
d	Distance
$\mathrm{d}/\mathrm{d}t$	Time derivative
$\mathbf{D}$	Material modulus matrix
$\mathbf{D}_c, \mathbf{D}_f$	Material modulus in coarse and fine scales
$\mathbf{D}_{\mathrm{loc}}$	Localisation modulus
$\mathrm{D}/\mathrm{D}t$	Material time derivative
$D_x^{\mathrm{b}}, D_x^{\mathrm{f}}$	Backward and forward finite difference approximations
E, E_i	Isotropic and orthotropic Young's modulus
E'	Material parameter
f_{t}	Uniaxial tensile strength
$f(r)$	Radial function
$\mathbf{f}$	Nodal force vector
$\mathbf{f}_i^r$	Nodal force components (classic and enriched)
$\mathbf{f}^{\mathrm{b}}$	Body force vector
$\mathbf{f}^{\mathrm{t}}$	External traction vector
$\mathbf{f}^{\mathrm{c}}$	Cohesive crack traction vector
$\mathbf{f}^{\mathrm{coh}}$	Cohesive nodal force vector
$\mathbf{f}^{\mathrm{ext}}$	External force vector
$\mathbf{f}_{\mathbf{u}}^{\mathrm{int}}$	Internal nodal force vector due to external loading
$\mathbf{f}_{\mathbf{a}}^{\mathrm{int}}$	Internal nodal force vector due to cohesive force
$F_l^i(x)$	Crack tip enrichment functions
$\mathbf{g}$	Applied gravitational body force
$g(\theta)$	Angular function for a crack tip kink problem
$g_j(\theta)$	Orthotropic crack tip enrichment functions
G	Shear modulus
G	Fracture energy release rate
G_1, G_2	Mode I and II fracture energy release rates
$G_{\mathrm{I}}^{\mathrm{dyn}}$	Dynamic mode I fracture energy release
$H(\xi)$	Heaviside function
H_{l}	Latent heat
i	Complex number, $i^2 = -1$
$\mathbf{J}$	Jacobian matrix
J	J integral

J^{act}	Actual J integral
J^{aux}	Auxiliary J integral
J_k	Mode k contour integral J
k_0	Dimensionless constant for the power hardening law
k_0, k_1, k_2, k_3, k_4	Constant coefficients
k_i	Conductivity coefficient for phase i
k_s, k_f	Thermal conductivity for solid and fluid phases
$k_{\text{n}}, k_{\text{t}}$	Normal/tangential interface properties
$\mathbf{K}$	Stiffness matrix
$\mathbf{K}_{\text{hom}}$	Homogenised stiffness matrix
$\mathbf{K}_{ij}^{rs}$	Stiffness matrix components
K	Stress intensity factor
K_{C}	Critical stress intensity factor
K_{eq}	Equivalent mixed mode stress intensity factor
$K_{\text{I}}, K_{\text{II}}, K_{\text{III}}$	Mode I , II and III stress intensity factors
$\bar{K}_{\text{I}}, \bar{K}_{\text{II}}$	Normalized mode I and mode II stress intensity factors
$K_{\text{I}}^{\text{aux}}, K_{\text{II}}^{\text{aux}}$	Auxiliary mode I and mode II stress intensity factors
$K_{\text{Ic}}, K_{\text{IIc}}$	Critical mode I and mode II stress intensity factors
$K_{\text{I}}^{\text{cohesion}}$	Cohesive mode I stress intensity factor
$K_{\text{I}}^{\text{crack}}$	Crack mode I stress intensity factor
$K_{\text{I}}^{\text{dyn}}$	Dynamic mode I stress intensity factor
l_e	Characteristic length
l_c	Characteristic length for crack propagaion
m	Number of enrichment functions
mt_{i}	Number of nodes to be enriched by crack tip enrichment functions
mf	Number of crack tip enrichment functions
M_j	Mach number
M	Interaction integral
M_0	Total mass
M_i	Lumped mass component
$\mathbf{M}_{ij}$	Mass matrix component
n	Power number for the plastic model (Section 2.6.4)
ng	Number of Gauss points
$\bar{n}$	Number of nodes within each moving least squares support domain
np	Number of independent domains of partition of unity
n_n	Number of nodes in a finite element
$\mathbf{n}$	Normal vector
$\mathbf{n}_{int}$	Normal vector to an internal interface
$\mathbf{N}_j$	Matrix of shape functions
N_j	Shape function
$\bar{N}_j$	New set of generalised finite element method shape functions
$p(\mathbf{x})$	Basis function
p	Hydrostatic pressure
$\bar{p}$	Predefined hydrostatic pressure
P_i	Loading condition i
q	Arbitrary smoothing function
q	Heat flux